CHAPTER 5

LINE MODEL AND PERFORMANCE

5.1 INTRODUCTION

In Chapter 4 the per-phase parameters of transmission lines were obtained. This chapter deals with the representation and performance of transmission lines under normal operating conditions. Transmission lines are represented by an equivalent model with appropriate circuit parameters on a "per-phase" basis. The terminal voltages are expressed from one line to neutral, the current for one phase and, thus, the three-phase system is reduced to an equivalent single-phase system.

The model used to calculate voltages, currents, and power flows depends on the length of the line. In this chapter the circuit parameters and voltage and current relations are first developed for "short" and "medium" lines. Problems relating to the regulation and losses of lines and their operation under conditions of fixed terminal voltages are then considered.

Next, long line theory is presented and expressions for voltage and current along the distributed line model are obtained. Propagation constant and characteristic impedance are defined, and it is demonstrated that the electrical power is being transmitted over the lines at approximately the speed of light. Since the terminal conditions at the two ends of the line are of primary importance, an equivalent

π model is developed for the long lines. Several *MATLAB* functions are developed for calculation of line parameters and performance. Finally, line compensations are discussed for improving the line performance for unloaded and loaded transmission lines.

5.2 SHORT LINE MODEL

Capacitance may often be ignored without much error if the lines are less than about 80 km (50 miles) long, or if the voltage is not over 69 kV. The short line model is obtained by multiplying the series impedance per unit length by the line length.

$$\begin{aligned} Z &= (r + j\omega L)\ell \\ &= R + jX \end{aligned} \tag{5.1}$$

where r and L are the per-phase resistance and inductance per unit length, respectively, and ℓ is the line length. The short line model on a per-phase basis is shown in Figure 5.1. V_S and I_S are the phase voltage and current at the sending end of the line, and V_R and I_R are the phase voltage and current at the receiving end of the line.

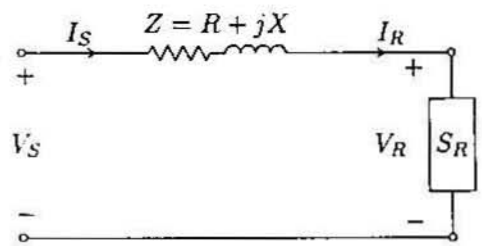

FIGURE 5.1
Short line model.

If a three-phase load with apparent power $S_{R(3\phi)}$ is connected at the end of the transmission line, the receiving end current is obtained by

$$I_R = \frac{S^*_{R(3\phi)}}{3V^*_R} \tag{5.2}$$

The phase voltage at the sending end is

$$V_S = V_R + ZI_R \tag{5.3}$$

and since the shunt capacitance is neglected, the sending end and the receiving end current are equal, i.e.,

$$I_S = I_R \tag{5.4}$$

The transmission line may be represented by a two-port network as shown in Figure 5.2, and the above equations can be written in terms of the generalized circuit constants commonly known as the $ABCD$ constants

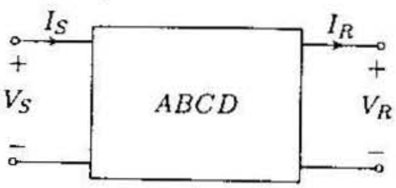

FIGURE 5.2
Two-port representation of a transmission line.

$$V_S = AV_R + BI_R \tag{5.5}$$

$$I_S = CV_R + DI_R \tag{5.6}$$

or in matrix form

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} \tag{5.7}$$

According to (5.3) and (5.4), for short line model

$$A = 1 \quad B = Z \quad C = 0 \quad D = 1 \tag{5.8}$$

Voltage regulation of the line may be defined as the percentage change in voltage at the receiving end of the line (expressed as percent of full-load voltage) in going from no-load to full-load.

$$\text{Percent } VR = \frac{|V_{R(NL)}| - |V_{R(FL)}|}{|V_{R(FL)}|} \times 100 \tag{5.9}$$

At no-load $I_R = 0$ and from (5.5)

$$V_{R(NL)} = \frac{V_S}{A} \tag{5.10}$$

For a short line, $A = 1$ and $V_{R(NL)} = V_S$. Voltage regulation is a measure of line voltage drop and depends on the load power factor. Voltage regulation will be

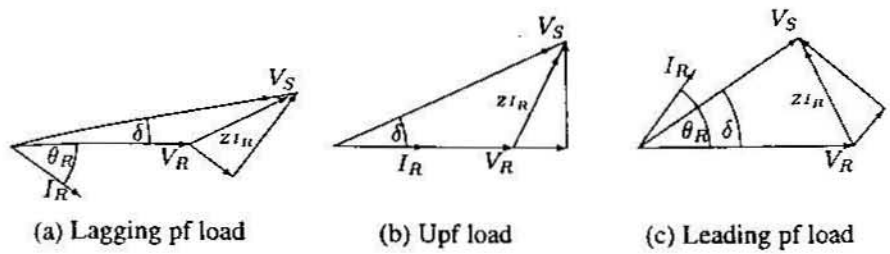

FIGURE 5.3
Phasor diagram for short line.

poorer at low lagging power factor loads. With capacitive loads, i.e., leading power factor loads, regulation may become negative. This is demonstrated by the phasor diagram of Figure 5.3.

Once the sending end voltage is calculated the sending-end power is obtained by

$$S_{S(3\phi)} = 3V_S I_S^* \tag{5.11}$$

The total line loss is then given by

$$S_{L(3\phi)} = S_{S(3\phi)} - S_{R(3\phi)} \tag{5.12}$$

and the transmission line efficiency is given by

$$\eta = \frac{P_{R(3\phi)}}{P_{S(3\phi)}} \tag{5.13}$$

where $P_{R(3\phi)}$ and $P_{S(3\phi)}$ are the total real power at the receiving end and sending end of the line, respectively.

Example 5.1

A 220-kV, three-phase transmission line is 40 km long. The resistance per phase is 0.15 Ω per km and the inductance per phase is 1.3263 mH per km. The shunt capacitance is negligible. Use the short line model to find the voltage and power at the sending end and the voltage regulation and efficiency when the line is supplying a three-phase load of

(a) 381 MVA at 0.8 power factor lagging at 220 kV.
(b) 381 MVA at 0.8 power factor leading at 220 kV.

(a) The series impedance per phase is

$$Z = (r + j\omega L)\ell = (0.15 + j2\pi \times 60 \times 1.3263 \times 10^{-3})40 = 6 + j20 \ \Omega$$

The receiving end voltage per phase is

$$V_R = \frac{220\angle 0^\circ}{\sqrt{3}} = 127\angle 0^\circ \ \text{kV}$$

The apparent power is

$$S_{R(3\phi)} = 381\angle\cos^{-1}0.8 = 381\angle 36.87^\circ = 304.8 + j228.6 \ \text{MVA}$$

The current per phase is given by

$$I_R = \frac{S^*_{R(3\phi)}}{3\,V^*_R} = \frac{381\angle -36.87^\circ \times 10^3}{3 \times 127\angle 0^\circ} = 1000\angle -36.87^\circ \ \text{A}$$

From (5.3) the sending end voltage is

$$V_S = V_R + ZI_R = 127\angle 0^\circ + (6 + j20)(1000\angle -36.87^\circ)(10^{-3})$$
$$= 144.33\angle 4.93^\circ \ \text{kV}$$

The sending end line-to-line voltage magnitude is

$$|V_{S(L-L)}| = \sqrt{3}\,|V_S| = 250 \ \text{kV}$$

The sending end power is

$$S_{S(3\phi)} = 3V_S I_S^* = 3 \times 144.33\angle 4.93 \times 1000\angle 36.87^\circ \times 10^{-3}$$
$$= 322.8 \ \text{MW} + j288.6 \ \text{Mvar}$$
$$= 433\angle 41.8^\circ \ \text{MVA}$$

Voltage regulation is

$$\text{Percent } VR = \frac{250 - 220}{220} \times 100 = 13.6\%$$

Transmission line efficiency is

$$\eta = \frac{P_{R(3\phi)}}{P_{S(3\phi)}} = \frac{304.8}{322.8} \times 100 = 94.4\%$$

(b) The current for 381 MVA with 0.8 leading power factor is

$$I_R = \frac{S^*_{R(3\phi)}}{3\,V^*_R} = \frac{381\angle 36.87^\circ \times 10^3}{3 \times 127\angle 0^\circ} = 1000\angle 36.87^\circ \ \text{A}$$

The sending end voltage is

$$V_S = V_R + ZI_R = 127\angle 0^\circ + (6 + j20)(1000\angle 36.87^\circ)(10^{-3})$$
$$= 121.39\angle 9.29^\circ \ \text{kV}$$

The sending end line-to-line voltage magnitude is

$$|V_{S(L-L)}| = \sqrt{3}\,V_S = 210.26 \ \text{kV}$$

The sending end power is

$$S_{S(3\phi)} = 3V_S I_S^* = 3 \times 121.39\angle 9.29 \times 1000\angle -36.87^\circ \times 10^{-3}$$
$$= 322.8 \ \text{MW} - j168.6 \ \text{Mvar}$$
$$= 364.18\angle -27.58^\circ \ \text{MVA}$$

Voltage regulation is

$$\text{Percent } VR = \frac{210.26 - 220}{220} \times 100 = -4.43\%$$

Transmission line efficiency is

$$\eta = \frac{P_{R(3\phi)}}{P_{S(3\phi)}} = \frac{304.8}{322.8} \times 100 = 94.4\%$$

5.3 MEDIUM LINE MODEL

As the length of line increases, the line charging current becomes appreciable and the shunt capacitance must be considered. Lines above 80 km (50 miles) and below 250 km (150 miles) in length are termed as *medium length lines*. For medium length lines, half of the shunt capacitance may be considered to be lumped at each end of the line. This is referred to as the *nominal* π model and is shown in Figure 5.4. Z is the total series impedance of the line given by (5.1), and Y is the total shunt admittance of the line given by

$$Y = (g + j\omega C)\ell \tag{5.14}$$

Under normal conditions, the shunt conductance per unit length, which represents the leakage current over the insulators and due to corona, is negligible and g is assumed to be zero. C is the line to neutral capacitance per km, and ℓ is the line length. The sending end voltage and current for the nominal π model are obtained as follows:

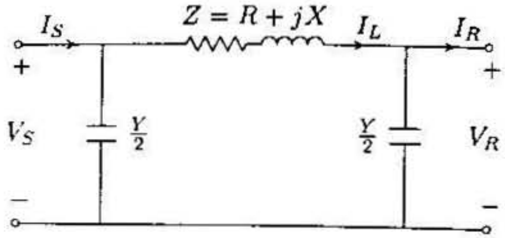

FIGURE 5.4
Nominal π model for medium length line.

From KCL the current in the series impedance designated by I_L is

$$I_L = I_R + \frac{Y}{2}V_R \tag{5.15}$$

From KVL the sending end voltage is

$$V_S = V_R + ZI_L \tag{5.16}$$

Substituting for I_L from (5.15), we obtain

$$V_S = \left(1 + \frac{ZY}{2}\right) V_R + ZI_R \tag{5.17}$$

The sending end current is

$$I_S = I_L + \frac{Y}{2}V_S \tag{5.18}$$

Substituting for I_L and V_S

$$I_S = Y\left(1 + \frac{ZY}{4}\right) V_R + \left(1 + \frac{ZY}{2}\right) I_R \tag{5.19}$$

Comparing (5.17) and (5.19) with (5.5) and (5.6), the $ABCD$ constants for the nominal π model are given by

$$A = \left(1 + \frac{ZY}{2}\right) \qquad B = Z \tag{5.20}$$

$$C = Y\left(1 + \frac{ZY}{4}\right) \qquad D = \left(1 + \frac{ZY}{2}\right) \tag{5.21}$$

In general, the $ABCD$ constants are complex and since the π model is a symmetrical two-port network, $A = D$. Furthermore, since we are dealing with a linear

passive, bilateral two-port network, the determinant of the transmission matrix in (5.7) is unity, i.e.,

$$AD - BC = 1 \tag{5.22}$$

Solving (5.7), the receiving end quantities can be expressed in terms of the sending end quantities by

$$\begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \begin{bmatrix} V_S \\ I_S \end{bmatrix} \tag{5.23}$$

Two *MATLAB* functions are written for computation of the transmission matrix. Function **[Z, Y, ABCD] = rlc2abcd(r, L, C, g, f, Length)** is used when resistance in ohm, inductance in mH and capacitance in μF per unit length are specified, and function **[Z, Y, ABCD] = zy2abcd(z, y, Length)** is used when series impedance in ohm and shunt admittance in siemens per unit length are specified. The above functions provide options for the nominal π model and the equivalent π model discussed in Section 5.4.

Example 5.2

A 345-kV, three-phase transmission line is 130 km long. The resistance per phase is 0.036 Ω per km and the inductance per phase is 0.8 mH per km. The shunt capacitance is 0.0112 μF per km. The receiving end load is 270 MVA with 0.8 power factor lagging at 325 kV. Use the medium line model to find the voltage and power at the sending end and the voltage regulation.

The function **[Z, Y, ABCD] = rlc2abcd(r, L, C, g, f, Length)** is used to obtain the transmission matrix of the line. The following commands

```
r = .036; g = 0; f = 60;
L = 0.8;            % milli-Henry
C = 0.0112;         % micro-Farad
Length = 130;  VR3ph = 325;
VR = VR3ph/sqrt(3) + j*0;   % kV (receiving end phase voltage)
[Z, Y, ABCD] = rlc2abcd(r, L, C, g, f, Length);
AR = acos(0.8);
SR = 270*(cos(AR) + j*sin(AR));  %   MVA (receiving end power)
IR = conj(SR)/(3*conj(VR));      %  kA (receiving end current)
VsIs = ABCD* [VR; IR];           %      column vector [Vs; Is]
Vs = VsIs(1);
Vs3ph = sqrt(3)*abs(Vs);         % kV(sending end L-L voltage)
Is = VsIs(2); Ism = 1000*abs(Is);%     A (sending end current)
pfs= cos(angle(Vs)- angle(Is));  %  (sending end power factor)
Ss = 3*Vs*conj(Is);              %      MVA (sending end power)
REG = (Vs3ph/abs(ABCD(1,1)) - VR3ph)/VR3ph *100;
```

```
fprintf(' Is = %g A', Ism), fprintf('  pf = %g', pfs)
fprintf(' Vs = %g L-L kV', Vs3ph)
fprintf(' Ps = %g MW', real(Ss)),
fprintf('  Qs = %g Mvar', imag(Ss))
fprintf(' Percent voltage Reg. = %g', REG)
```

result in

```
Enter 1 for Medium line or 2 for long line → 1
Nominal π model
Z = 4.68 + j 39.2071 ohms
Y = 0 + j 0.000548899 siemens
```

$$\text{ABCD} = \begin{bmatrix} 0.98924 & + \; j\; 0.0012844 & 4.68 & + \; j\; 39.207 \\ -3.5251\text{e-}07 & + \; j\; 0.00054595 & 0.98924 & + \; j\; 0.0012844 \end{bmatrix}$$

```
Is = 421.132 A        pf = 0.869657
Vs = 345.002 L-L kV
Ps = 218.851 MW     Qs = 124.23 Mvar
Percent voltage Reg. = 7.30913
```

Example 5.3

A 345-kV, three-phase transmission line is 130 km long. The series impedance is $z = 0.036 + j0.3$ Ω per phase per km, and the shunt admittance is $y = j4.22 \times 10^{-6}$ siemens per phase per km. The sending end voltage is 345 kV, and the sending end current is 400 A at 0.95 power factor lagging. Use the medium line model to find the voltage, current and power at the receiving end and the voltage regulation.

The function **[Z, Y, ABCD] = zy2abcd(z, y, Length)** is used to obtain the transmission matrix of the line. The following commands

```
z = .036 + j* 0.3;  y = j*4.22/1000000;  Length = 130;
Vs3ph = 345;  Ism = 0.4;  %kA;
As = -acos(0.95);
Vs = Vs3ph/sqrt(3) + j*0;      % kV (sending end phase voltage)
Is = Ism*(cos(As) + j*sin(As));
[Z,Y, ABCD] = zy2abcd(z, y, Length);
VrIr = inv(ABCD)* [Vs; Is];     %         column vector [Vr; Ir]
Vr = VrIr(1);
Vr3ph = sqrt(3)*abs(Vr);        % kV(receiving end L-L voltage)
Ir = VrIr(2); Irm = 1000*abs(Ir);  % A (receiving end current)
pfr= cos(angle(Vr)- angle(Ir));  %(receiving end power factor)
Sr = 3*Vr*conj(Ir);                      % MVA (receiving end power)
```



```
REG = (Vs3ph/abs(ABCD(1,1)) - Vr3ph)/Vr3ph *100;
fprintf(' Ir = %g A', Irm), fprintf('  pf = %g', pfr)
fprintf(' Vr = %g L-L kV', Vr3ph)
fprintf(' Pr = %g MW', real(Sr))
fprintf('  Qr = %g Mvar', imag(Sr))
fprintf(' Percent voltage Reg. = %g', REG)
```

result in

```
Enter 1 for Medium line or 2 for long line → 1
Nominal π model
Z = 4.68 + j 39 ohms
Y = 0 + j 0.0005486 siemens
```

$$\text{ABCD} = \begin{bmatrix} 0.9893 & + \; j\; 0.0012837 & 4.68 & + \; j\; 39 \\ -3.5213\text{e-}07 & + \; j\; 0.00054565 & 0.9893 & + \; j\; 0.0012837 \end{bmatrix}$$

```
Ir = 441.832 A        pf = 0.88750
Vr = 330.68 L-L kV
Pr = 224.592 MW     Qr = 116.612 Mvar
Percent voltage Reg. = 5.45863
```

5.4 LONG LINE MODEL

For the short and medium length lines reasonably accurate models were obtained by assuming the line parameters to be lumped. For lines 250 km (150 miles) and longer and for a more accurate solution the exact effect of the distributed parameters must be considered. In this section expressions for voltage and current at any point on the line are derived. Then, based on these equations an equivalent π model is obtained for the long line. Figure 5.5 shows one phase of a distributed line of length ℓ km.

The series impedance per unit length is shown by the lowercase letter z, and the shunt admittance per phase is shown by the lowercase letter y, where $z = r + j\omega L$ and $y = g + j\omega C$. Consider a small segment of line Δx at a distance x from the receiving end of the line. The phasor voltages and currents on both sides of this segment are shown as a function of distance. From Kirchhoff's voltage law

$$V(x + \Delta x) = V(x) + z\,\Delta x\, I(x) \tag{5.24}$$

or

$$\frac{V(x + \Delta x) - V(x)}{\Delta x} = z\, I(x) \tag{5.25}$$

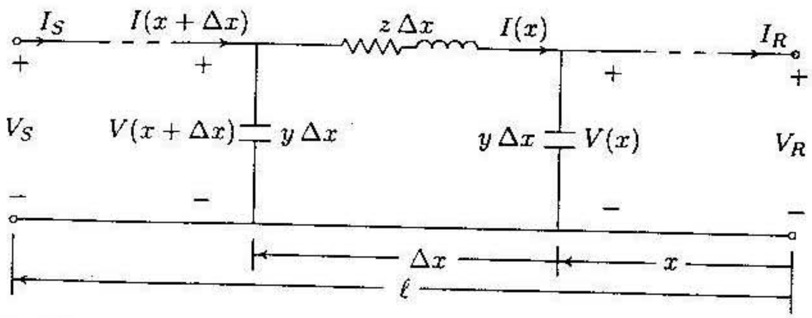

FIGURE 5.5
Long line with distributed parameters.

Taking the limit as $\Delta x \to 0$, we have

$$\frac{dV(x)}{dx} = z\,I(x) \tag{5.26}$$

Also, from Kirchhoff's current law

$$I(x+\Delta x) = I(x) + y\,\Delta x\,V(x+\Delta x) \tag{5.27}$$

or

$$\frac{I(x+\Delta x) - I(x)}{\Delta x} = y\,V(x+\Delta x) \tag{5.28}$$

Taking the limit as $\Delta x \to 0$, we have

$$\frac{dI(x)}{dx} = y\,V(x) \tag{5.29}$$

Differentiating (5.26) and substituting from (5.29), we get

$$\begin{aligned}\frac{d^2V(x)}{dx^2} &= z\,\frac{dI(x)}{dx} \\ &= zy\,V(x)\end{aligned} \tag{5.30}$$

Let

$$\gamma^2 = zy \tag{5.31}$$

The following second-order differential equation will result.

$$\frac{d^2V(x)}{dx^2} - \gamma^2\,V(x) = 0 \tag{5.32}$$

The solution of the above equation is

$$V(x) = A_1 e^{\gamma x} + A_2 e^{-\gamma x} \tag{5.33}$$

where γ, known as the *propagation constant*, is a complex expression given by (5.31) or

$$\gamma = \alpha + j\beta = \sqrt{zy} = \sqrt{(r + j\omega L)(g + j\omega C)} \tag{5.34}$$

The real part α is known as the *attenuation constant*, and the imaginary component β is known as the *phase constant*. β is measured in radian per unit length.

From (5.26), the current is

$$\begin{aligned}I(x) = \frac{1}{z}\frac{dV(x)}{dx} &= \frac{\gamma}{z}(A_1 e^{\gamma x} - A_2 e^{-\gamma x}) \\ &= \sqrt{\frac{y}{z}}(A_1 e^{\gamma x} - A_2 e^{-\gamma x})\end{aligned} \tag{5.35}$$

or

$$I(x) = \frac{1}{Z_c}(A_1 e^{\gamma x} - A_2 e^{-\gamma x}) \tag{5.36}$$

where Z_c is known as the *characteristic impedance*, given by

$$Z_c = \sqrt{\frac{z}{y}} \tag{5.37}$$

To find the constants A_1 and A_2 we note that when $x = 0$, $V(x) = V_R$, and $I(x) = I_R$. From (5.33) and (5.36) these constants are found to be

$$\begin{aligned}A_1 &= \frac{V_R + Z_c I_R}{2} \\ A_2 &= \frac{V_R - Z_c I_R}{2}\end{aligned} \tag{5.38}$$

Upon substitution in (5.33) and (5.36), the general expressions for voltage and current along a long transmission line become

$$V(x) = \frac{V_R + Z_c I_R}{2} e^{\gamma x} + \frac{V_R - Z_c I_R}{2} e^{-\gamma x} \tag{5.39}$$

$$I(x) = \frac{\frac{V_R}{Z_c} + I_R}{2} e^{\gamma x} - \frac{\frac{V_R}{Z_c} - I_R}{2} e^{-\gamma x} \tag{5.40}$$

The equations for voltage and currents can be rearranged as follows:

$$V(x) = \frac{e^{\gamma x} + e^{-\gamma x}}{2} V_R + Z_c \frac{e^{\gamma x} - e^{-\gamma x}}{2} I_R \qquad (5.41)$$

$$I(x) = \frac{1}{Z_c} \frac{e^{\gamma x} - e^{-\gamma x}}{2} V_R + \frac{e^{\gamma x} + e^{-\gamma x}}{2} I_R \qquad (5.42)$$

Recognizing the hyperbolic functions sinh, and cosh, the above equations are written as follows:

$$V(x) = \cosh \gamma x \, V_R + Z_c \sinh \gamma x \, I_R \qquad (5.43)$$

$$I(x) = \frac{1}{Z_c} \sinh \gamma x \, V_R + \cosh \gamma x \, I_R \qquad (5.44)$$

We are particularly interested in the relation between the sending end and the receiving end of the line. Setting $x = \ell$, $V(\ell) = V_s$ and $I(\ell) = I_s$, the result is

$$V_s = \cosh \gamma\ell \, V_R + Z_c \sinh \gamma\ell \, I_R \qquad (5.45)$$

$$I_s = \frac{1}{Z_c} \sinh \gamma\ell \, V_R + \cosh \gamma\ell \, I_R \qquad (5.46)$$

Rewriting the above equations in terms of the $ABCD$ constants as before, we have

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} \qquad (5.47)$$

where

$$A = \cosh \gamma\ell \qquad B = Z_c \sinh \gamma\ell \qquad (5.48)$$

$$C = \frac{1}{Z_c} \sinh \gamma\ell \qquad D = \cosh \gamma\ell \qquad (5.49)$$

Note that, as before, $A = D$ and $AD - BC = 1$.

It is now possible to find an accurate equivalent π model, shown in Figure 5.6, to replace the $ABCD$ constants of the two-port network. Similar to the expressions (5.17) and (5.19) obtained for the nominal π, for the equivalent π model we have

$$V_S = \left(1 + \frac{Z'Y'}{2}\right) V_R + Z' I_R \qquad (5.50)$$

$$I_S = Y'\left(1 + \frac{Z'Y'}{4}\right) V_R + \left(1 + \frac{Z'Y'}{2}\right) I_R \qquad (5.51)$$

Comparing (5.50) and (5.51) with (5.45) and (5.46), respectively, and making use of the identity

$$\tanh \frac{\gamma\ell}{2} = \frac{\cosh \gamma\ell - 1}{\sinh \gamma\ell} \qquad (5.52)$$

the parameters of the equivalent π model are obtained.

$$Z' = Z_c \sinh \gamma\ell = Z \frac{\sinh \gamma\ell}{\gamma\ell} \qquad (5.53)$$

$$\frac{Y'}{2} = \frac{1}{Z_c} \tanh \frac{\gamma\ell}{2} = \frac{Y}{2} \frac{\tanh \gamma\ell/2}{\gamma\ell/2} \qquad (5.54)$$

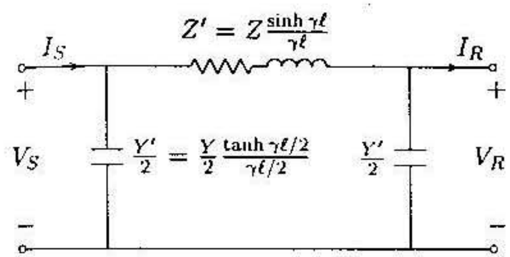

FIGURE 5.6
Equivalent π model for long length line.

The functions **[Z, Y, ABCD] = rlc2abcd(r, L, C, g, f, Length)** and **[Z, Y, ABCD] = zy2abcd(z, y, Length)** with option 2 can be used for the evaluation of the transmission matrix and the equivalent π parameters. However, Example 5.4 shows how these hyperbolic functions can be evaluated easily with simple *MATLAB* commands.

Example 5.4

A 500-kV, three-phase transmission line is 250 km long. The series impedance is $z = 0.045 + j0.4\ \Omega$ per phase per km and the shunt admittance is $y = j4 \times 10^{-6}$ siemens per phase per km. Evaluate the equivalent π model and the transmission matrix

The following commands

```
z = 0.045 + j*.4;     y = j*4.0/1000000; Length = 250;
gamma = sqrt(z*y);  Zc = sqrt(z/y);
A = cosh(gamma*Length);  B = Zc*sinh(gamma*Length);
C = 1/Zc * sinh(gamma*Length);  D = A;
ABCD = [A  B; C  D]
Z = B; Y = 2/Zc * tanh(gamma*Length/2)
```

result in

```
ABCD =
        0.9504 + 0.0055i   10.8778 +98.3624i
       -0.0000 + 0.0010i    0.9504 + 0.0055i
Z =
       10.8778 +98.3624i
Y =
        0.0000 + 0.0010i
```

5.5 VOLTAGE AND CURRENT WAVES

The rms expression for the phasor value of voltage at any point along the line is given by (5.33). Substituting $\alpha + j\beta$ for γ, the phasor voltage is

$$V(x) = A_1 e^{\alpha x} e^{j\beta x} + A_2 e^{-\alpha x} e^{-j\beta x}$$

Transforming from phasor domain to time domain, the instantaneous voltage as a function of t and x becomes

$$v(t,x) = \sqrt{2}\,\Re\, A_1 e^{\alpha x} e^{j(\omega t+\beta x)} + \sqrt{2}\,\Re\, A_2 e^{-\alpha x} e^{j(\omega t-\beta x)} \tag{5.55}$$

As x increases (moving away from the receiving end), the first term becomes larger because of $e^{\alpha x}$ and is called the *incident wave*. The second term becomes smaller because of $e^{-\alpha x}$ and is called the *reflected wave*. At any point along the line, voltage is the sum of these two components.

$$v(t,x) = v_1(t,x) + v_2(t,x) \tag{5.56}$$

where

$$v_1(t,x) = \sqrt{2}\, A_1 e^{\alpha x} \cos(\omega t + \beta x) \tag{5.57}$$

$$v_2(t,x) = \sqrt{2}\, A_2 e^{-\alpha x} \cos(\omega t - \beta x) \tag{5.58}$$

As the current expression is similar to the voltage, the current can also be considered as the sum of incident and reflected current waves.

Equations (5.57) and (5.58) behave like traveling waves as we move along the line. This is similar to the disturbance in the water at some sending point. To see this, consider the reflected wave $v_2(t,x)$ and imagine that we ride along with the wave. To observe the instantaneous value, for example the peak amplitude requires that

$$\omega t - \beta x = 2K\pi \quad \text{or} \quad x = \frac{\omega}{\beta}t - \frac{2K\pi}{\beta}$$

Thus, to keep up with the wave and observe the peak amplitude we must travel with the speed

$$\frac{dx}{dt} = \frac{\omega}{\beta} \tag{5.59}$$

Thus, the velocity of propagation is given by

$$v = \frac{\omega}{\beta} = \frac{2\pi f}{\beta} \tag{5.60}$$

The wavelength λ or distance x on the wave which results in a phase shift of 2π radian is

$$\beta\lambda = 2\pi$$

or

$$\lambda = \frac{2\pi}{\beta} \tag{5.61}$$

When line losses are neglected, i.e., when $g = 0$ and $r = 0$, the real part of the propagation constant $\alpha = 0$, and from (5.34) the phase constant becomes

$$\beta = \omega\sqrt{LC} \tag{5.62}$$

Also, the characteristic impedance is purely resistive and (5.37) becomes

$$Z_c = \sqrt{\frac{L}{C}} \tag{5.63}$$

which is commonly referred to as the *surge impedance*. Substituting for β in (5.60) and (5.61), for a lossless line the velocity of propagation and the wavelength become

$$v = \frac{1}{\sqrt{LC}} \tag{5.64}$$

$$\lambda = \frac{1}{f\sqrt{LC}} \tag{5.65}$$

The expressions for the inductance per unit length L and capacitance per unit length C of a transmission line were derived in Chapter 4, given by (4.58) and (4.91). When the internal flux linkage of a conductor is neglected $GMR_L = GMR_C$, and upon substitution (5.64) and (5.65) become

$$v \simeq \frac{1}{\sqrt{\mu_0\varepsilon_0}} \tag{5.66}$$

$$\lambda \simeq \frac{1}{f\sqrt{\mu_0\varepsilon_0}} \tag{5.67}$$

Substituting for $\mu_0 = 4\pi \times 10^{-7}$ and $\varepsilon_0 = 8.85 \times 10^{-12}$, the velocity of the wave is obtained to be approximately 3×10^8 m/sec, i.e., the velocity of light. At 60 Hz, the wavelength is 5000 km. Similarly, substituting for L and C in (5.63), we have

$$Z_c \simeq \frac{1}{2\pi}\sqrt{\frac{\mu_0}{\varepsilon_0}}\ln\frac{GMD}{GMR_c}$$
$$\simeq 60\ln\frac{GMD}{GMR_c} \tag{5.68}$$

For typical transmission lines the surge impedance varies from approximately 400Ω for 69-kV lines down to around 250 Ω for double-circuit 765-kV transmission lines.

For a lossless line $\gamma = j\beta$ and the hyperbolic functions $\cosh\gamma x = \cosh j\beta x = \cos\beta x$ and $\sinh\gamma x = \sinh j\beta x = j\sin\beta x$, the equations for the rms voltage and current along the line, given by (5.43) and (5.44), become

$$V(x) = \cos\beta x\, V_R + jZ_c \sin\beta x\, I_R \tag{5.69}$$
$$I(x) = j\frac{1}{Z_c}\sin\beta x\, V_R + \cos\beta x\, I_R \tag{5.70}$$

At the sending end $x = \ell$

$$V_S = \cos\beta\ell\, V_R + jZ_c \sin\beta\ell\, I_R \tag{5.71}$$
$$I_S = j\frac{1}{Z_c}\sin\beta\ell\, V_R + \cos\beta\ell\, I_R \tag{5.72}$$

For hand calculation it is easier to use (5.71) and (5.72), and for more accurate calculations (5.47) through (5.49) can be used in *MATLAB*. The terminal conditions are readily obtained from the above equations. For example, for the open-circuited line $I_R = 0$, and from (5.71) the no-load receiving end voltage is

$$V_{R(nl)} = \frac{V_S}{\cos\beta\ell} \tag{5.73}$$

At no-load, the line current is entirely due to the line charging capacitive current and the receiving end voltage is higher than the sending end voltage. This is evident from (5.73), which shows that as the line length increases $\beta\ell$ increases and $\cos\beta\ell$ decreases, resulting in a higher no-load receiving end voltage.

For a solid short circuit at the receiving end, $V_R = 0$ and (5.71) and (5.72) reduce to

$$V_S = jZ_c \sin\beta\ell\, I_R \tag{5.74}$$
$$I_S = \cos\beta\ell\, I_R \tag{5.75}$$

The above equations can be used to find the short circuit currents at both ends of the line.

5.6 SURGE IMPEDANCE LOADING

When the line is loaded by being terminated with an impedance equal to its characteristic impedance, the receiving end current is

$$I_R = \frac{V_R}{Z_c} \tag{5.76}$$

For a lossless line Z_c is purely resistive. The load corresponding to the surge impedance at rated voltage is known as the ***surge impedance loading*** (SIL), given by

$$SIL = 3V_R I_R^* = \frac{3|V_R|^2}{Z_c} \tag{5.77}$$

Since $V_R = V_{Lrated}/\sqrt{3}$, SIL in MW becomes

$$SIL = \frac{(kV_{Lrated})^2}{Z_c}\ \text{MW} \tag{5.78}$$

Substituting for I_R in (5.69) and V_R in (5.70) will result in

$$V(x) = (\cos\beta x + j\sin\beta x)V_R \quad \text{or} \quad V(x) = V_R\angle\beta x \tag{5.79}$$
$$I(x) = (\cos\beta x + j\sin\beta x)I_R \quad \text{or} \quad I(x) = I_R\angle\beta x \tag{5.80}$$

Equations (5.79) and (5.80) show that in a lossless line under surge impedance loading the voltage and current at any point along the line are constant in magnitude and are equal to their sending end values. Since Z_c has no reactive component, there is no reactive power in the line, $Q_S = Q_R = 0$. This indicates that for SIL, the reactive losses in the line inductance are exactly offset by reactive power supplied by the shunt capacitance or $\omega L|I_R|^2 = \omega C|V_R|^2$. From this relation, we find that $Z_c = V_R/I_R = \sqrt{L/C}$, which verifies the result in (5.63). SIL for typical transmission lines varies from approximately 150 MW for 230-kV lines to about 2000 MW for 765-kV lines. SIL is a useful measure of transmission line capacity as it indicates a loading where the line's reactive requirements are small. For loads significantly above SIL, shunt capacitors may be needed to minimize voltage drop along the line, while for light loads significantly below SIL, shunt inductors may be needed. Generally the transmission line full-load is much higher than SIL. The voltage profile for various loading conditions is illustrated in Figure 5.11 (page 182) in Example 5.9(h).

Example 5.5

A three-phase, 60-Hz, 500-kV transmission line is 300 km long. The line inductance is 0.97 mH/km per phase and its capacitance is 0.0115 μF/km per phase. Assume a lossless line.

(a) Determine the line phase constant β, the surge impedance Z_C, velocity of propagation v and the line wavelength λ.
(b) The receiving end rated load is 800 MW, 0.8 power factor lagging at 500 kV. Determine the sending end quantities and the voltage regulation.

(a) For a lossless line, from (5.62) we have

$$\beta = \omega\sqrt{LC} = 2\pi \times 60\sqrt{0.97 \times 0.0115 \times 10^{-9}} = 0.001259 \text{ rad/km}$$

and from (5.63)

$$Z_c = \sqrt{\frac{L}{C}} = \sqrt{\frac{0.97 \times 10^{-3}}{0.0115 \times 10^{-6}}} = 290.43\ \Omega$$

Velocity of propagation is

$$v = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.97 \times 0.0115 \times 10^{-9}}} = 2.994 \times 10^5 \text{ km/s}$$

and the line wavelength is

$$\lambda = \frac{v}{f} = \frac{1}{60}(2.994 \times 10^5) = 4990 \text{ km}$$

(b) $\beta\ell = 0.001259 \times 300 = 0.3777 \text{ rad} = 21.641^\circ$

The receiving end voltage per phase is

$$V_R = \frac{500\angle 0^\circ}{\sqrt{3}} = 288.675\angle 0^\circ \text{ kV}$$

The receiving end apparent power is

$$S_{R(3\phi)} = \frac{800}{0.8}\angle\cos^{-1} 0.8 = 1000\angle 36.87^\circ = 800 + j600 \text{ MVA}$$

The receiving end current per phase is given by

$$I_R = \frac{S^*_{R(3\phi)}}{3V^*_R} = \frac{1000\angle -36.87^\circ \times 10^3}{3 \times 288.675\angle 0^\circ} = 1154.7\angle -36.87^\circ \text{ A}$$

From (5.71) the sending end voltage is

$$\begin{aligned} V_S &= \cos\beta\ell\, V_R + jZ_c \sin\beta\ell\, I_R \\ &= (0.9295)288.675\angle 0^\circ + j(290.43)(0.3688)(1154.7\angle -36.87^\circ)(10^{-3}) \\ &= 356.53\angle 16.1^\circ \text{ kV} \end{aligned}$$

The sending end line-to-line voltage magnitude is

$$|V_{S(L-L)}| = \sqrt{3}\,|V_S| = 617.53 \text{ kV}$$

From (5.72) the sending end current is

$$\begin{aligned} I_S &= j\frac{1}{Z_c}\sin\beta\ell\, V_R + \cos\beta\ell\, I_R \\ &= j\frac{1}{290.43}(0.3688)(288.675\angle 0^\circ)(10^3) + (0.9295)(1154.7\angle -36.87^\circ) \\ &= 902.3\angle -17.9^\circ \text{ A} \end{aligned}$$

The sending end power is

$$\begin{aligned} S_{S(3\phi)} = 3V_S I_S^* &= 3 \times 356.53\angle 16.1 \times 902.3\angle -17.9^\circ \times 10^{-3} \\ &= 800 \text{ MW} + j539.672 \text{ Mvar} \\ &= 965.1\angle 34^\circ \text{ MVA} \end{aligned}$$

Voltage regulation is

$$\text{Percent } VR = \frac{356.53/0.9295 - 288.675}{288.675} \times 100 = 32.87\%$$

The line performance of the above transmission line including the line resistance is obtained in Example 5.9 using the **lineperf** program. When a line is operating at the rated load, the exact solution results in $V_{S(L-L)} = 623.5\angle 15.57^\circ$ kV, and $I_s = 903.1\angle -17.7^\circ$ A. This shows that the lossless assumption yields acceptable results and is suitable for hand calculation.

5.7 COMPLEX POWER FLOW THROUGH TRANSMISSION LINES

Specific expressions for the complex power flow on a line may be obtained in terms of the sending end and receiving end voltage magnitudes and phase angles and the $ABCD$ constants. Consider Figure 5.2 where the terminal relations are given by (5.5) and (5.6). Expressing the $ABCD$ constants in polar form as $A = |A|\angle\theta_A$,

$B = |B|\angle\theta_B$, the sending end voltage as $V_S = |V_S|\angle\delta$, and the receiving end voltage as reference $V_R = |V_R|\angle 0$, from (5.5) I_R can be written as

$$I_R = \frac{|V_S|\angle\delta - |A|\angle\theta_A|V_R|\angle 0}{|B|\angle\theta_B} = \frac{|V_S|}{|B|}\angle\delta - \theta_B - \frac{|A||V_R|}{|B|}\angle\theta_A - \theta_B \tag{5.81}$$

The receiving end complex power is

$$S_{R(3\phi)} = P_{R(3\phi)} + jQ_{R(3\phi)} = 3V_R I_R^* \tag{5.82}$$

Substituting for I_R from (5.81), we have

$$S_{R(3\phi)} = 3\frac{|V_S||V_R|}{|B|}\angle\theta_B - \delta - 3\frac{|A||V_R|^2}{|B|}\angle\theta_B - \theta_A \tag{5.83}$$

or in terms of the line-to-line voltages, we have

$$S_{R(3\phi)} = \frac{|V_{S(L-L)}||V_{R(L-L)}|}{|B|}\angle\theta_B - \delta - \frac{|A||V_{R(L-L)}|^2}{|B|}\angle\theta_B - \theta_A \tag{5.84}$$

The real and reactive power at the receiving end of the line are

$$P_{R(3\phi)} = \frac{|V_{S(L-L)}||V_{R(L-L)}|}{|B|}\cos(\theta_B - \delta) - \frac{|A||V_{R(L-L)}|^2}{|B|}\cos(\theta_B - \theta_A) \tag{5.85}$$

$$Q_{R(3\phi)} = \frac{|V_{S(L-L)}||V_{R(L-L)}|}{|B|}\sin(\theta_B - \delta) - \frac{|A||V_{R(L-L)}|^2}{|B|}\sin(\theta_B - \theta_A) \tag{5.86}$$

The sending end power is

$$S_{S(3\phi)} = P_{S(3\phi)} + jQ_{S(3\phi)} = 3V_S I_S^* \tag{5.87}$$

From (5.23), I_S can be written as

$$I_S = \frac{|A|\angle\theta_A|V_S|\angle\delta - |V_R|\angle 0}{|B|\angle\theta_B} \tag{5.88}$$

Substituting for I_S in (5.87) yields

$$P_{S(3\phi)} = \frac{|A||V_{S(L-L)}|^2}{|B|}\cos(\theta_B - \theta_A) - \frac{|V_{S(L-L)}||V_{R(L-L)}|}{|B|}\cos(\theta_B + \delta) \tag{5.89}$$

$$Q_{S(3\phi)} = \frac{|A||V_{S(L-L)}|^2}{|B|}\sin(\theta_B - \theta_A) - \frac{|V_{S(L-L)}||V_{R(L-L)}|}{|B|}\sin(\theta_B + \delta) \tag{5.90}$$

The real and reactive transmission line losses are

$$P_{L(3\phi)} = P_{S(3\phi)} - P_{R(3\phi)} \tag{5.91}$$

$$Q_{L(3\phi)} = Q_{S(3\phi)} - Q_{R(3\phi)} \tag{5.92}$$

The locus of all points obtained by plotting $Q_{R(3\phi)}$ versus $P_{R(3\phi)}$ for fixed line voltages and varying load angle δ is a circle known as the *receiving end power circle diagram*. A family of such circles with fixed receiving end voltage and varying sending end voltage is extremely useful in assessing the performance characteristics of the transmission line. A function called **pwrcirc(ABCD)** is developed for the construction of the receiving end power circle diagram, and its use is demonstrated in Example 5.9(g).

For a lossless line $B = jX'$, $\theta_A = 0$, $\theta_B = 90°$, and $A = \cos\beta\ell$, and the real power transferred over the line is given by

$$P_{3\phi} = \frac{|V_{S(L-L)}||V_{R(L-L)}|}{X'}\sin\delta \tag{5.93}$$

and the receiving end reactive power is

$$Q_{R3\phi} = \frac{|V_{S(L-L)}||V_{R(L-L)}|}{X'}\cos\delta - \frac{|V_{R(L-L)}|^2}{X'}\cos\beta\ell \tag{5.94}$$

For a given system operating at constant voltage, the power transferred is proportional to the sine of the power angle δ. As the load increases, δ increases. For a lossless line, the maximum power that can be transmitted under stable steady-state condition occurs for an angle of 90°. However, a transmission system with its connected synchronous machines must also be able to withstand, without loss of stability, sudden changes in generation, load, and faults. To assure an adequate margin of stability, the practical operating load angle is usually limited to 35 to 45°.

5.8 POWER TRANSMISSION CAPABILITY

The power handling ability of a line is limited by the thermal loading limit and the stability limit. The increase in the conductor temperature, due to the real power loss, stretches the conductors. This will increase the sag between transmission towers. At higher temperatures this may result in irreversible stretching. The thermal limit is specified by the current-carrying capacity of the conductor and is available in the manufacturer's data. If the current-carrying capacity is denoted by $I_{thermal}$, the thermal loading limit of a line is

$$S_{thermal} = 3V_{\phi rated}I_{thermal} \tag{5.95}$$

The expression for real power transfer over the line for a lossless line is given by (5.93). The theoretical maximum power transfer is when $\delta = 90°$. The practical operating load angle for the line alone is limited to no more than 30 to 45°. This is because of the generator and transformer reactances which, when added to the line, will result in a larger δ for a given load. For planning and other purposes, it is very useful to express the power transfer formula in terms of SIL, and construct the line loadability curve. For a lossless line $X' = Z_c \sin \beta\ell$, and (5.93) may be written as

$$P_{3\phi} = \left(\frac{|V_{S(L-L)}|}{V_{rated}}\right)\left(\frac{|V_{R(L-L)}|}{V_{rated}}\right)\left(\frac{V_{rated}^2}{Z_c}\right)\frac{\sin\delta}{\sin\beta\ell} \tag{5.96}$$

The first two terms within parenthesis are the per-unit voltages denoted by V_{Spu} and V_{Rpu}, and the third term is recognized as SIL. Equation (5.96) may be written as

$$P_{3\phi} = \frac{|V_{Spu}||V_{Rpu}|SIL}{\sin\beta\ell}\sin\delta$$
$$= \frac{|V_{Spu}||V_{Rpu}|SIL}{\sin(\frac{2\pi\ell}{\lambda})}\sin\delta \tag{5.97}$$

The function **loadabil(L, C, f)** obtains the loadability curve and thermal limit curve of the line. The loadability curve as obtained in Figure 5.12 (page 182) for Example 5.9(i) shows that for short and medium lines the thermal limit dictates the maximum power transfer. Whereas, for longer lines the limit is set by the practical line loadability curve. As we see in the next section, for longer lines it may be necessary to use series capacitors in order to increase the power transfer over the line.

Example 5.6

A three-phase power of 700-MW is to be transmitted to a substation located 315 km from the source of power. For a preliminary line design assume the following parameters:

$V_S = 1.0$ per unit, $V_R = 0.9$ per unit, $\lambda = 5000$ km, $Z_c = 320\ \Omega$, and $\delta = 36.87°$

(a) Based on the practical line loadability equation determine a nominal voltage level for the transmission line.

(b) For the transmission voltage level obtained in (a) calculate the theoretical maximum power that can be transferred by the transmission line.

(a) From (5.61), the line phase constant is

$$\beta\ell = \frac{2\pi}{\lambda}\ell \text{ rad}$$
$$= \frac{360}{\lambda}\ell = \frac{360}{5000}(315) = 22.68°$$

From the practical line loadability given by (5.97), we have

$$700 = \frac{(1.0)(0.9)(SIL)}{\sin(22.68°)}\sin(36.87°)$$

Thus

$$SIL = 499.83 \text{ MW}$$

From (5.78)

$$kV_L = \sqrt{(Z_c)(SIL)} = \sqrt{(320)(499.83)} = 400 \text{ kV}$$

(b) The equivalent line reactance for a lossless line is given by

$$X' = Z_c \sin\beta\ell = 320\sin(22.68) = 123.39\ \Omega$$

For a lossless line, the maximum power that can be transmitted under steady state condition occurs for a load angle of 90°. Thus, from (5.93), assuming $|V_S| = 1.0$ pu and $|V_R| = 0.9$ pu, the theoretical maximum power is

$$P_{3\phi(max)} = \frac{(400)(0.9)(400)}{123.39}(1) = 1167 \text{ MW}$$

5.9 LINE COMPENSATION

We have noted that a transmission line loaded to its surge impedance loading has no net reactive power flow into or out of the line and will have approximately a flat voltage profile along its length. On long transmission lines, light loads appreciably less than SIL result in a rise of voltage at the receiving end, and heavy loads appreciably greater than SIL will produce a large dip in voltage. The voltage profile of a long line for various loading conditions is shown in Figure 5.11 (page 182). Shunt reactors are widely used to reduce high voltages under light load or open line conditions. If the transmission system is heavily loaded, shunt capacitors, static var control, and synchronous condensers are used to improve voltage, increase power transfer, and improve the system stability.

5.9.1 SHUNT REACTORS

Shunt reactors are applied to compensate for the undesirable voltage effects associated with line capacitance. The amount of reactor compensation required on a transmission line to maintain the receiving end voltage at a specified value can be obtained as follows.

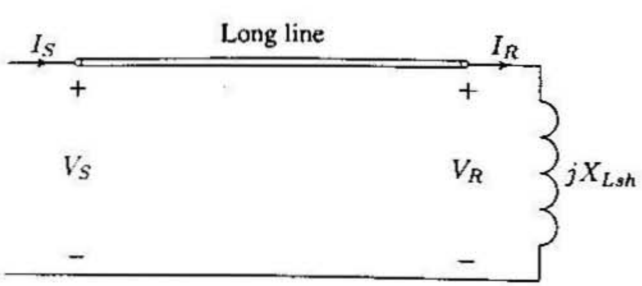

FIGURE 5.7
Shunt reactor compensation.

Consider a reactor of reactance X_{Lsh}, connected at the receiving end of a long transmission line as shown in Figure 5.7. The receiving end current is

$$I_R = \frac{V_R}{jX_{Lsh}} \tag{5.98}$$

Substituting I_R into (5.71) results in

$$V_S = V_R(\cos\beta\ell + \frac{Z_c}{X_{Lsh}}\sin\beta\ell)$$

Note that V_S and V_R are in phase, which is consistent with the fact that no real power is being transmitted over the line. Solving for X_{Lsh} yields

$$X_{Lsh} = \frac{\sin\beta\ell}{\frac{V_S}{V_R} - \cos\beta\ell} Z_c \tag{5.99}$$

For $V_S = V_R$, the required inductor reactance is

$$X_{Lsh} = \frac{\sin\beta\ell}{1 - \cos\beta\ell} Z_c \tag{5.100}$$

To find the relation between I_S and I_R, we substitute for V_R from (5.98) into (5.72)

$$I_S = \left(-\frac{1}{Z_c}\sin\beta\ell\, X_{Lsh} + \cos\beta\ell\right) I_R$$

Substituting for X_{Lsh} from (5.100) for the case when $V_S = V_R$ results in

$$I_S = -I_R \tag{5.101}$$

With one reactor only at the receiving end, the voltage profile will not be uniform, and the maximum rise occurs at the midspan. It is left as an exercise to show that for $V_S = V_R$, the voltage at the midspan is given by

$$V_m = \frac{V_R}{\cos\frac{\beta\ell}{2}} \tag{5.102}$$

Also, the current at the midspan is zero. The function **openline(ABCD)** is used to find the receiving end voltage of an open line and to determine the Mvar of the reactor required to maintain the no-load receiving end voltage at a specified value. Example 5.9(d) illustrates the reactor compensation. Installing reactors at both ends of the line will improve the voltage profile and reduce the tension at midspan.

Example 5.7

For the transmission line of Example 5.5:
(a) Calculate the receiving end voltage when line is terminated in an open circuit and is energized with 500 kV at the sending end.
(b) Determine the reactance and the Mvar of a three-phase shunt reactor to be installed at the receiving end to keep the no-load receiving end voltage at the rated value.

(a) The line is energized with 500 kV at the sending end. The sending end voltage per phase is

$$V_S = \frac{500\angle 0^\circ}{\sqrt{3}} = 288.675 \text{ kV}$$

From Example 5.5, $Z_c = 290.43$ and $\beta\ell = 21.641°$.

When the line is open $I_R = 0$ and from (5.71) the no-load receiving end voltage is given by

$$V_{R(nl)} = \frac{V_S}{\cos\beta\ell} = \frac{288.675}{0.9295} = 310.57 \text{ kV}$$

The no-load receiving end line-to-line voltage is

$$V_{R(L-L)(nl)} = \sqrt{3}\, V_{R(nl)} = 537.9 \text{ kV}$$

(b) For $V_S = V_R$, the required inductor reactance given by (5.100) is

$$X_{Lsh} = \frac{\sin(21.641°)}{1 - \cos(21.641°)}(290.43) = 1519.5 \ \Omega$$

The three-phase shunt reactor rating is

$$Q_{3\phi} = \frac{(kV_{Lrated})^2}{X_{Lsh}} = \frac{(500)^2}{1519.5} = 164.53 \text{ Mvar}$$

5.9.2 SHUNT CAPACITOR COMPENSATION

Shunt capacitors are used for lagging power factor circuits created by heavy loads. The effect is to supply the requisite reactive power to maintain the receiving end voltage at a satisfactory level. Capacitors are connected either directly to a bus bar or to the tertiary winding of a main transformer and are disposed along the route to minimize the losses and voltage drops. Given V_S and V_R, (5.85) and (5.86) can be used conveniently to compute the required capacitor Mvar at the receiving end for a specified load. A function called **shntcomp(ABCD)** is developed for this purpose, and its use is demonstrated in Example 5.9(f).

5.9.3 SERIES CAPACITOR COMPENSATION

Series capacitors are connected in series with the line, usually located at the mid-point, and are used to reduce the series reactance between the load and the supply point. This results in improved transient and steady-state stability, more economical loading, and minimum voltage dip on load buses. Series capacitors have the good characteristics that their reactive power production varies concurrently with the line loading. Studies have shown that the addition of series capacitors on EHV transmission lines can more than double the transient stability load limit of long lines at a fraction of the cost of a new transmission line.

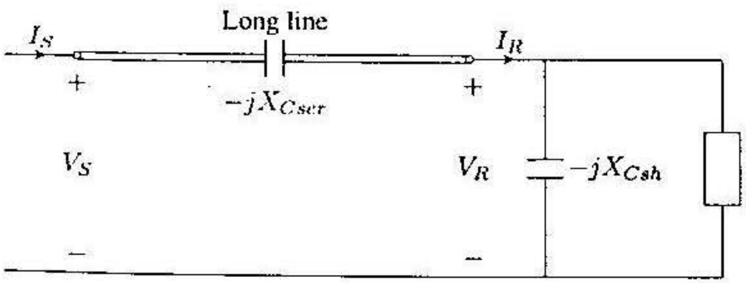

FIGURE 5.8
Shunt and series capacitor compensation.

With the series capacitor switched on as shown in Figure 5.8, from (5.93), the power transfer over the line for a lossless line becomes

$$P_{3\phi} = \frac{|V_{S(L-L)}||V_{R(L-L)}|}{X' - X_{Cser}} \sin\delta \qquad (5.103)$$

Where X_{Cser} is the series capacitor reactance. The ratio X_{Cser}/X' expressed as a percentage is usually referred to as the *percentage compensation*. The percentage compensation is in the range of 25 to 70 percent.

One major drawback with series capacitor compensation is that special protective devices are required to protect the capacitors and bypass the high current produced when a short circuit occurs. Also, inclusion of series capacitors establishes a resonant circuit that can oscillate at a frequency below the normal synchronous frequency when stimulated by a disturbance. This phenomenon is referred to as *subsynchronous resonance* (SSR). If the synchronous frequency minus the electrical resonant frequency approaches the frequency of one of the turbine-generator natural torsional modes, considerable damage to the turbine-generator may result. If L' is the lumped line inductance corrected for the effect of distribution and C_{ser} is the capacitance of the series capacitor, the subsynchronous resonant frequency is

$$f_r = f_s\sqrt{\frac{1}{L'C_{ser}}} \qquad (5.104)$$

where f_s is the synchronous frequency. The function **sercomp(ABCD)** can be used to obtain the line performance for a specified percentage compensation. Finally, when line is compensated with both series and shunt capacitors, for the specified terminal voltages, the function **srshcomp(ABCD)** is used to obtain the line performance and the required shunt capacitor. These compensations are also demonstrated in Example 5.9(f).

Example 5.8

The transmission line in Example 5.5 supplies a load of 1000 MVA, 0.8 power factor lagging at 500 kV.

(a) Determine the Mvar and the capacitance of the shunt capacitors to be installed at the receiving end to keep the receiving end voltage at 500 kV when the line is energized with 500 kV at the sending end.

(b) Only series capacitors are installed at the midpoint of the line providing 40 percent compensation. Find the sending end voltage and voltage regulation.

(a) From Example 5.5, $Z_c = 290.43$ and $\beta\ell = 21.641°$. Thus, the equivalent line reactance for a lossless line is given by

$$X' = Z_c \sin\beta\ell = 290.43\sin(21.641°) = 107.11 \ \Omega$$

The receiving end power is

$$S_{R(3\phi)} = 1000\angle\cos^{-1}(0.8) = 800 + j600 \ \text{MVA}$$

For the above operating condition, the power angle δ is obtained from (5.93)

$$800 = \frac{(500)(500)}{107.11}\sin\delta$$

which results in $\delta = 20.044°$. Using the approximate relation given by (5.94), the net reactive power at the receiving end is

$$Q_{R(3\phi)} = \frac{(500)(500)}{107.11}\cos(20.044°) - \frac{(500)^2}{107.11}\cos(21.641°) = 23.15 \text{ Mvar}$$

Thus, the required capacitor Mvar is $S_C = j23.15 - j600 = -j576.85$

The capacitive reactance is given by

$$X_C = \frac{|V_L|^2}{S_C^*} = \frac{(500)^2}{j576.85} = -j433.38 \;\Omega$$

or

$$C = \frac{10^6}{2\pi(60)(433.38)} = 6.1 \;\mu\text{F}$$

The shunt compensation for the above transmission line including the line resistance is obtained in Example 5.9(f) using the **lineperf** program. The exact solution results in 613.8 Mvar for capacitor reactive power as compared to 576.85 Mvar obtained from the approximate formula for the lossless line. This represents approximately an error of 6 percent.

(b) For 40 percent compensation, the series capacitor reactance per phase is

$$X_{ser} = 0.4X' = 0.4(107.1) = 42.84 \;\Omega$$

The new equivalent π circuit parameters are given by

$$Z' = j(X' - X_{ser}) = j(107.1 - 42.84) = j64.26 \;\Omega$$

$$Y' = = j\frac{2}{Z_c}\tan(\beta\ell/2) = j\frac{2}{290.43}\tan(21.641°/2) = j0.001316 \text{ siemens}$$

The new B constant is $B = j64.26$ and the new A-constant is given by

$$A = 1 + \frac{Z'Y'}{2} = 1 + \frac{(j64.26)(j0.001316)}{2} = 0.9577$$

The receiving end voltage per phase is

$$V_R = \frac{500}{\sqrt{3}} = 288.675 \text{ kV}$$

and the receiving end current is

$$I_R = \frac{S^*_{R(3\phi)}}{3V_R^*} = \frac{1000\angle -36.87°}{3 \times 288.675\angle 0°} = 1.1547\angle -36.87° \text{ kA}$$

Thus, the sending end voltage is

$$\begin{aligned} V_S = AV_R + BI_R &= 0.9577 \times 288.675 + j64.26 \times 1.1547\angle -36.87° \\ &= 326.4\angle 10.47° \text{ kV} \end{aligned}$$

and the line-to-line voltage magnitude is $|V_{S(L-L)}| = \sqrt{3}\,V_S = 565.4$ kV. Voltage regulation is

$$\text{Percent } VR = \frac{565.4/0.958 - 500}{500} \times 100 = 18\%$$

The exact solution obtained in Example 5.9(f) results in $V_{S(L-L)} = 571.9$ kV. This represents an error of 1.0 percent.

5.10 LINE PERFORMANCE PROGRAM

A program called **lineperf** is developed for the complete analysis and compensation of a transmission line. The command **lineperf** displays a menu with five options for the computation of the parameters of the π models and the transmission constants. Selection of these options will call upon the following functions.

[Z, Y, ABCD] = rlc2abcd(r, L, C, g, f, Length) computes and returns the π model parameters and the transmission constants when r in ohm, L in mH, and C in μF per unit length, frequency, and line length are specified.

[Z, Y, ABCD] = zy2abcd(z, y, Length) computes and returns the π model parameters and the transmission constants when impedance and admittance per unit length are specified.

[Z, Y, ABCD] = pi2abcd(Z, Y) returns the ABCD constants when the π model parameters are specified.

[Z, Y, ABCD] = abcd2pi(A, B, C) returns the π model parameters when the transmission constants are specified.

[L , C] = gmd2lc computes and returns the inductance and capacitance per phase when the line configuration and conductor dimensions are specified.

[r, L, C, f] = abcd2rlc(ABCD) returns the line parameters per unit length and frequency when the transmission constants are specified.

Any of the above functions can be used independently when the arguments of the functions are defined in the *MATLAB* environment. If the above functions are typed without the parenthesis and the arguments, the user will be prompted to enter the required data. Next the **lineperf** loads the program **listmenu** which displays a list of eight options for transmission line analysis and compensation. Selection of these options will call upon the following functions.

givensr(ABCD) prompts the user to enter V_R, P_R and Q_R. This function computes V_S, P_S, Q_S, line losses, voltage regulation, and transmission efficiency.

givenss(ABCD) prompts the user to enter V_S, P_S and Q_S. This function computes V_R, P_R, Q_R, line losses, voltage regulation, and transmission efficiency.

givenzl(ABCD) prompts the user to enter V_R and the load impedance. This function computes V_S, P_S, Q_S, line losses, voltage regulation, and transmission efficiency.

openline(ABCD) prompts the user to enter V_S. This function computes V_R for the open-ended line. Also, the reactance and the Mvar of the necessary reactor to maintain the receiving end voltage at a specified value are obtained. In addition, the function plots the voltage profile of the line.

shcktlin(ABCD) prompts the user to enter V_S. This function computes the current at both ends of the line for a solid short circuit at the receiving end.

Option 6 is for capacitive compensation and calls upon **compmenu** which displays three options. Selection of these options will call upon the following functions.

shntcomp(ABCD) prompts the user to enter V_S, P_R, Q_R and the desired V_R. This function computes the capacitance and the Mvar of the shunt capacitor bank to be installed at the receiving end in order to maintain the specified V_R. Then, V_S, P_S, Q_S, line losses, voltage regulation, and transmission efficiency are found.

sercomp(ABCD) prompts the user to enter V_R, P_R, Q_R, power, and the percentage compensation (i.e., $X_{Cser}/X_{line} \times 100$). This function computes the Mvar of the specified series capacitor and V_S, P_S, Q_S, line losses, voltage regulation, and transmission efficiency for the compensated line.

srshcomp(ABCD) prompts the user to enter V_S, P_R, Q_R, the desired V_R and the percentage series capacitor compensation. This function computes the capacitance and the Mvar of a shunt capacitor to be installed at the receiving end in order to maintain the specified V_R. Also, V_S, P_S, Q_S, line losses, voltage regulation, and transmission efficiency are obtained for the compensated line.

Option 7 loads the **pwrcirc(ABCD)** which prompts for the receiving end voltage. This function constructs the receiving end power circle diagram for various values of V_S from V_R up to $1.3V_R$.

Option 8 calls upon **profmenu** which displays two options. Selection of these options will call upon the following functions:

vprofile(r, L, C, f) prompts the user to enter V_S, rated MVA, power factor, V_R, P_R, and Q_R. This function displays a graph consisting of voltage profiles for line length up to $1/8$ of the line wavelength for the following cases: open-ended line, line terminated in SIL, short-circuited line, and full-load.

loadabil(L, C, f) prompts the user for V_S, V_R, rated line voltage, and current-carrying capacity of the line. This function displays a graph consisting of the practical line loadability curve for $\delta = 30°$, the theoretical stability limit curve, and the thermal limit. This function assumes a lossless line and the plots are obtained for a line length up to $1/4$ of the line wavelength.

Any of the above functions can be used independently when the arguments of the functions are defined in the *MATLAB* environment. The $ABCD$ constant is entered as a matrix. If the above functions are typed without the parenthesis and the arguments, the user will be prompted to enter the required data.

Example 5.9

A three-phase, 60-Hz, 550-kV transmission line is 300 km long. The line parameters per phase per unit length are found to be

$$r = 0.016\ \Omega/\text{km} \quad L = 0.97\ \text{mH/km} \quad C = 0.0115\ \mu\text{F/km}$$

(a) Determine the line performance when load at the receiving end is 800 MW, 0.8 power factor lagging at 500 kV.

The command:

```
lineperf
```

displays the following menu

```
Type of parameters for input                  Select
Parameters per unit length
r (Ω), g (siemens), L (mH), C (μF)              1

Complex z and y per unit length
r + j*x (Ω), g + j*b (siemens)                  2

Nominal π or Eq. π model                        3

A, B, C, D constants                            4

Conductor configuration and dimension           5

To quit                                         0
Select number of menu → 1
Enter line length = 300
Enter frequency in Hz = 60
Enter line resistance/phase in Ω/unit length, r = 0.016
Enter line inductance in mH per unit length, L = 0.97
Enter line capacitance in μF per unit length, C = .0115
Enter line conductance in siemens per unit length, g = 0
Enter 1 for medium line or 2 for long line → 2

Equivalent π model
Z' = 4.57414 + j 107.119 ohms
Y' = 6.9638e-07 + j 0.00131631 siemens
Zc = 290.496 + j -6.35214 ohms
αℓ = 0.00826172 neper βℓ = 0.377825 radian = 21.6478°
```

$$\text{ABCD.} = \begin{bmatrix} 0.9295 & + & j0.0030478 & 4.5741 & + & j107.12 \\ -1.3341e-06 & + & j0.0012699 & 0.9295 & + & j0.0030478 \end{bmatrix}$$

At this point the program **listmenu** is automatically loaded and displays the following menu.

```
        Transmission line performance
               Analysis                            Select

 To calculate sending end quantities
 for specified receiving end MW, Mvar                1
```



```
To calculate receiving end quantities
for specified sending end MW, Mvar                   2

To calculate sending end quantities
when load impedance is specified                     3

Open-end line and reactive compensation              4

Short-circuited line                                 5

Capacitive compensation                              6

Receiving end circle diagram                         7

Loadability curve and voltage profile                8


To quit                                              0

Select number of menu → 1

Enter receiving end line-line voltage kV = 500
Enter receiving end voltage phase angle° = 0
Enter receiving end 3-phase power MW = 800
Enter receiving end 3-phase reactive power
(+ for lagging and - for leading power factor) Mvar = 600

Line performance for specified receiving end quantities

Vr = 500 kV (L-L) at 0°
Pr = 800 MW Qr = 600 Mvar
Ir = 1154.7 A at -36.8699° PFr = 0.8 lagging
Vs = 623.511 kV (L-L) at 15.5762°
Is = 903.113 A at -17.6996°, PFs = 0.836039 lagging
Ps = 815.404 MW,     Qs = 535.129 Mvar
PL = 15.4040 MW,     QL = -64.871 Mvar
Percent Voltage Regulation = 34.1597
Transmission line efficiency = 98.1108
```

At the end of this analysis the **listmenu** (Analysis Menu) is displayed.

(b) Determine the receiving end quantities and the line performance when 600 MW and 400 Mvar are being transmitted at 525 kV from the sending end.

Selecting option 2 of the **listmenu** results in

```
Enter sending end line-line voltage kV = 525
Enter sending end voltage phase angle° = 0
Enter sending end 3-phase power MW = 600
Enter sending end 3-phase reactive power
(+ for lagging and - for leading power factor) Mvar = 400

Line performance for specified sending end quantities

Vs = 525 kV (L-L) at 0°
Ps = 600 MW,      Qs = 400 Mvar
Is = 793.016 A at -33.6901°, PFs = 0.83205 lagging
Vr = 417.954 kV (L-L) at -16.3044°
Ir = 1002.6 A at -52.16° PFr = 0.810496 lagging
Pr = 588.261 MW,      Qr = 425.136 Mvar
PL = 11.7390 MW,      QL = -25.136 Mvar
Percent Voltage Regulation = 35.1383
Transmission line efficiency = 98.0435
```

(c) Determine the sending end quantities and the line performance when the receiving end load impedance is 290 Ω at 500 kV.

Selecting option 3 of the listmenu results in

```
Enter receiving end line-line voltage kV = 500
Enter receiving end voltage phase angle° = 0
Enter sending end complex load impedance 290 + j * 0

Line performance for specified load impedance

Vr = 500 kV (L-L) at 0°
Ir = 995.431 A at 0° PFr = 1
Pr = 862.069 MW,      Qr = 0 Mvar
Vs = 507.996 kV (L-L) at 21.5037°
Is = 995.995 A at 21.7842°, PFs = 0.999988 leading
Ps = 876.341 MW Qs = -4.290 Mvar
PL = 14.272 MW QL = -4.290 Mvar
Percent Voltage Regulation = 9.30464
Transmission line efficiency = 98.3714
```

(d) Find the receiving end voltage when the line is terminated in an open circuit and is energized with 500 kV at the sending end. Also, determine the reactance and the Mvar of a three-phase shunt reactor to be installed at the receiving end in order to limit the no-load receiving end voltage to 500 kV.

Selecting option 4 of the **listmenu** results in

```
Enter sending end line-line voltage kV = 500
Enter sending end voltage phase angle° = 0

Open line and shunt reactor compensation

Vs = 500 kV (L-L) at 0°
Vr = 537.92 kV (L-L) at -0.00327893°
Is = 394.394 A at 89.8723°, PFs = 0.0022284 leading
Desired no load receiving end voltage = 500 kV
Shunt reactor reactance = 1519.4 Ω
Shunt reactor rating = 164.538 Mvar
```

The voltage profile for the uncompensated and the compensated line is also found as shown in Figure 5.9.

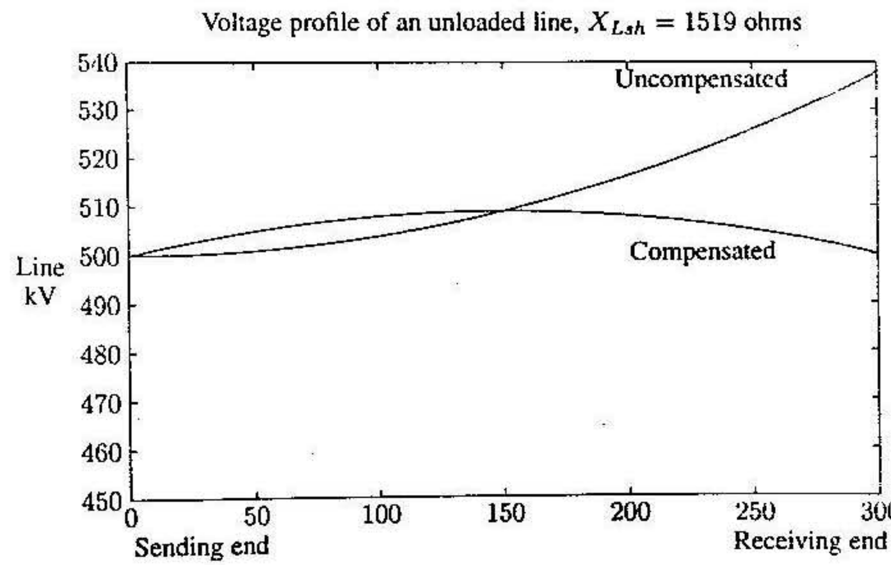

FIGURE 5.9
Compensated and uncompensated voltage profile of open-ended line.

(e) Find the receiving end and the sending end currents when the line is terminated in a short circuit.

Selecting option 5 of the **listmenu** results in

```
Enter sending end line-line voltage kV = 500
Enter sending end voltage phase angle° = 0

Line short-circuited at the receiving end

Vs = 500 kV (L-L) at 0°
Ir = 2692.45 A at -87.5549°
Is = 2502.65 A at -87.367°
```

(f) The line loading in part (a) resulted in a voltage regulation of 34.16 percent, which is unacceptably high. To improve the line performance, the line is compensated with series and shunt capacitors. For the loading condition in (a):

(1) Determine the Mvar and the capacitance of the shunt capacitors to be installed at the receiving end to keep the receiving end voltage at 500 kV when the line is energized with 500 kV at the sending end.

Selecting option 6 will display the **compmenu** as follows:

Capacitive compensation Analysis	Select
Shunt capacitive compensation	1
Series capacitive compensation	2
Series and shunt capacitive compensation	3
To quit	0

Selecting option 1 of the **compmenu** results in

```
Enter sending end line-line voltage kV = 500
Enter desired receiving end line-line voltage kV = 500
Enter receiving end voltage phase angle° = 0
Enter receiving end 3-phase power MW = 800
Enter receiving end 3-phase reactive power
(+ for lagging and - for leading power factor) Mvar = 600
```



```
Shunt capacitive compensation

Vs = 500 kV (L-L) at 20.2479°
Vr = 500 kV (L-L) at 0°
Pload = 800 MW,    Qload = 600 Mvar
Load current = 1154.7 A at -36.8699°, PFl = 0.8 lagging
Required shunt capacitor: 407.267 Ω, 6.51314 μF,613.849 Mvar
Shunt capacitor current = 708.811 A at 90°
Pr = 800.000 MW,    Qr = -13.849 Mvar
Ir = 923.899 A at 0.991732°, PFr = 0.99985 leading
Is = 940.306 A at 24.121° PFs = 0.997716 leading
Ps = 812.469 MW,    Qs = -55.006 Mvar
PL = 12.469 MW,    QL = -41.158 Mvar
Percent Voltage Regulation = 7.58405
Transmission line efficiency = 98.4653
```

(2) Determine the line performance when the line is compensated by series capacitors for 40 percent compensation with the load condition in (a) at 500 kV.

Selecting option 2 of the **compmenu** results in

```
Enter receiving end line-line voltage kV = 500
Enter receiving end voltage phase angle° = 0
Enter receiving end 3-phase power MW = 800
Enter receiving end 3-phase reactive power
(+ for lagging and - for leading power factor) Mvar = 600
Enter percent compensation for series capacitor
(Recommended range 25 to 75% of the line reactance) = 40

Series capacitor compensation

Vr = 500 kV (L-L) at 0°
Pr = 800 MW,    Qr = 600 Mvar
Required series capacitor: 42.8476 Ω, 61.9074 μF, 47.4047 Mvar
Subsynchronous resonant frequency = 37.9473 Hz
Ir = 1154.7 A at -36.8699°, PFr = 0.8 lagging
Vs = 571.904 kV (L-L) at 9.95438°
Is = 932.258 A at -18.044°, PFs = 0.882961 lagging
Ps = 815.383 MW,    Qs = 433.517 Mvar
PL = 15.383 MW,    QL = -166.483 Mvar
Percent Voltage Regulation = 19.4322
Transmission line efficiency = 98.1134
```

(3) The line has 40 percent series capacitor compensation and supplies the load in (a). Determine the Mvar and the capacitance of the shunt capacitors to be installed at the receiving end to keep the receiving end voltage at 500 kV when line is energized with 500 kV at the sending end.

Selecting option 3 of the **compmenu** results in

```
Enter sending end line-line voltage kV = 500
Enter desired receiving end line-line voltage kV = 500
Enter receiving end voltage phase angle° = 0
Enter receiving end 3-phase power MW = 800
Enter receiving end 3-phase reactive power
(+ for lagging and - for leading power factor) Mvar = 600
Enter percent compensation for series capacitor
(Recommended range 25 to 75% of the line reactance) = 40
```

```
Series and shunt capacitor compensation

Vs = 500 kV (L-L) at 12.0224°
Vr = 500 kV (L-L) at 0°
Pload = 800 MW,      Qload = 600 Mvar
Load current = 1154.7 A at -36.8699°, PFl = 0.8 lagging
Required shunt capacitor: 432.736 Ω, 6.1298 μF, 577.72 Mvar
Shunt capacitor current = 667.093 A at 90°
Required series capacitor: 42.8476 Ω, 61.9074 μF,37.7274 Mvar
Subsynchronous resonant frequency = 37.9473 Hz
Pr = 800 MW,      Qr = 22.2804 Mvar
Ir = 924.119 A at -1.5953°, PFr = 0.999612 lagging
Is = 951.165 A at 21.5977°, PFs = 0.986068 leading
Ps = 812.257 MW,      Qs = -137.023 Mvar
PL = 12.257 MW,      QL = -159.304 Mvar
Percent Voltage Regulation = 4.41619
Transmission line efficiency = 98.491
```

(g) Construct the receiving end circle diagram.

Selecting option 7 of the **listmenu** results in

```
Enter receiving end line-line voltage kV = 500
```

A plot of the receiving end circle diagram is obtained as shown in Figure 5.10.

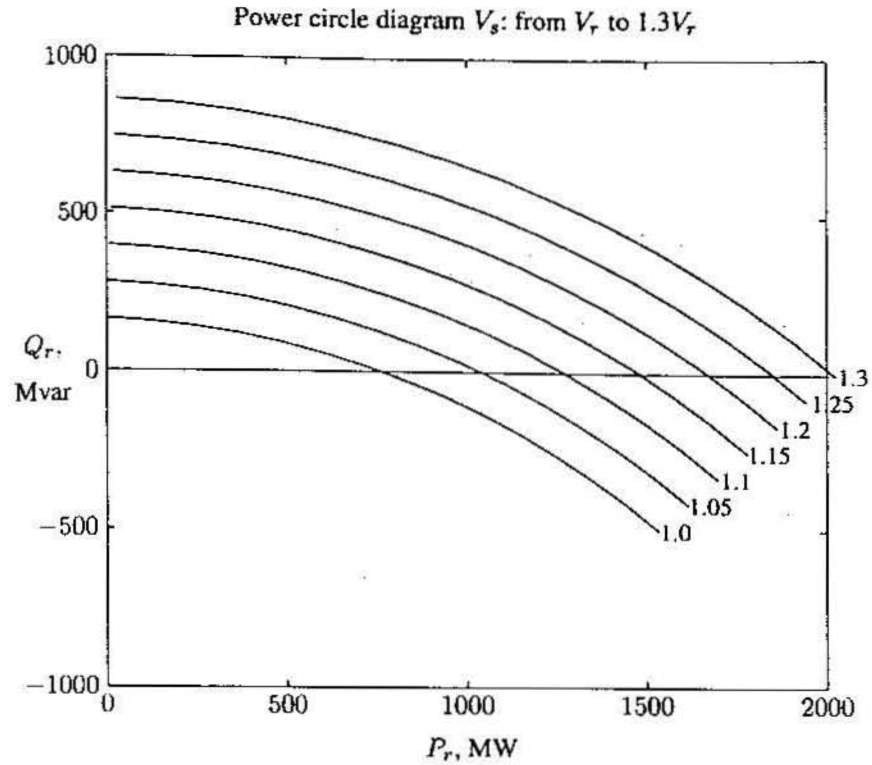

FIGURE 5.10
Receiving end circle diagram.

(h) Determine the line voltage profile for the following cases: no-load, rated load, line terminated in the SIL, and short-circuited line.

Selecting option 8 of the **listmenu** results in

```
          Voltage profile and line loadability
                     Analysis                      Select

Voltage profile curves                               1
Line loadability curve                               2
To quit                                              0
```

Selecting option 1 of the **profmenu** results in

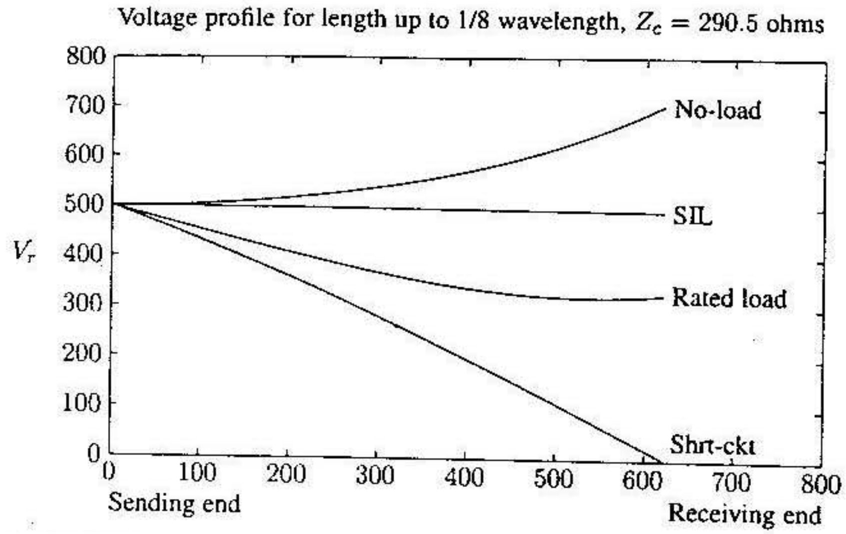

FIGURE 5.11
Voltage profile for length up to 1/8 wavelength.

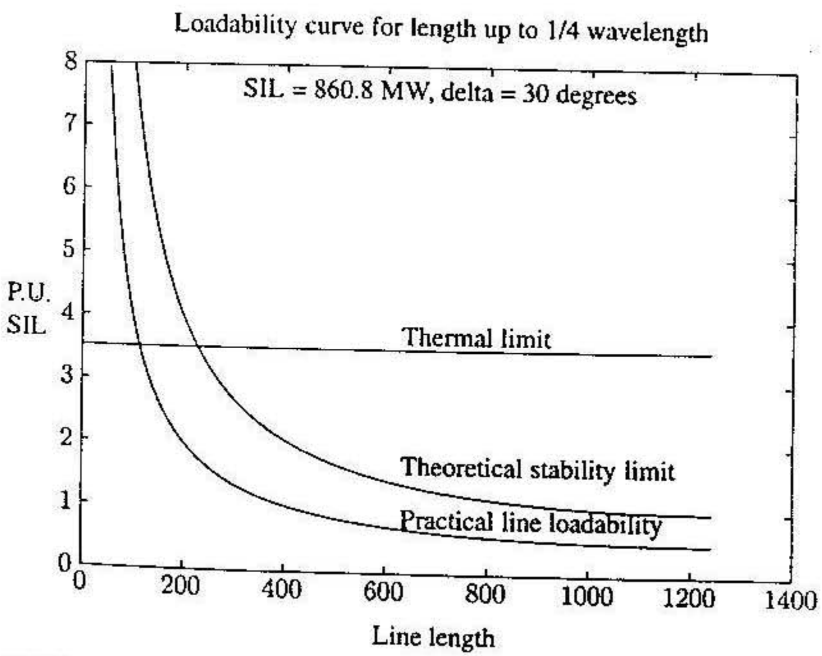

FIGURE 5.12
Line loadability curve for length up to 1/4 wavelength.


```
Enter sending end line-line voltage kV = 500
Enter rated sending end power, MVA = 1000
Enter power factor = 0.8
```

A plot of the voltage profile is obtained as shown in Figure 5.11 (page 182).

(i) Obtain the line loadability curves.
Selecting option 2 of the **profmenu** results in

```
Enter sending end line-line voltage kV = 500
Enter receiving end line-line voltage kV = 500
Enter rated line-line voltage kV = 500
Enter line current-carrying capacity, Amp/phase = 3500
```

The line loadability curve is obtained as shown in Figure 5.12 (page 182).

PROBLEMS

5.1. A 69-kV, three-phase short transmission line is 16 km long. The line has a per phase series impedance of $0.125 + j0.4375$ Ω per km. Determine the sending end voltage, voltage regulation, the sending end power, and the transmission efficiency when the line delivers

(a) 70 MVA, 0.8 lagging power factor at 64 kV.
(b) 120 MW, unity power factor at 64 kV.

Use **lineperf** program to verify your results.

5.2. Shunt capacitors are installed at the receiving end to improve the line performance of Problem 5.1. The line delivers 70 MVA, 0.8 lagging power factor at 64 kV. Determine the total Mvar and the capacitance per phase of the Y-connected capacitors when the sending end voltage is

(a) 69 kV.
(b) 64 kV.

Hint: Use (5.85) and (5.86) to compute the power angle δ and the receiving end reactive power.

(c) Use **lineperf** to obtain the compensated line performance.

5.3. A 230-kV, three-phase transmission line has a per phase series impedance of $z = 0.05 + j0.45$ Ω per km and a per phase shunt admittance of $y = j3.4 \times 10^{-6}$ siemens per km. The line is 80 km long. Using the nominal π model, determine

(a) The transmission line ABCD constants.

Find the sending end voltage and current, voltage regulation, the sending end power and the transmission efficiency when the line delivers

(b) 200 MVA, 0.8 lagging power factor at 220 kV.
(c) 306 MW, unity power factor at 220 kV.

Use **lineperf** program to verify your results.

5.4. Shunt capacitors are installed at the receiving end to improve the line performance of Problem 5.3. The line delivers 200 MVA, 0.8 lagging power factor at 220 kV.

(a) Determine the total Mvar and the capacitance per phase of the Y-connected capacitors when the sending end voltage is 220 kV. *Hint*: Use (5.85) and (5.86) to compute the power angle δ and the receiving end reactive power.
(b) Use **lineperf** to obtain the compensated line performance.

5.5. A three-phase, 345-kV, 60-Hz transposed line is composed of two ACSR, 1,113,000-cmil, 45/7 Bluejay conductors per phase with flat horizontal spacing of 11 m. The conductors have a diameter of 3.195 cm and a GMR of 1.268 cm. The bundle spacing is 45 cm. The resistance of each conductor in the bundle is 0.0538 Ω per km and the line conductance is negligible. The line is 150 km long. Using the nominal π model, determine the ABCD constant of the line. Use **lineperf** and option 5 to verify your results.

5.6. The ABCD constants of a three-phase, 345-kV transmission line are

$$A = D = 0.98182 + j0.0012447$$
$$B = 4.035 + j58.947$$
$$C = j0.00061137$$

The line delivers 400 MVA at 0.8 lagging power factor at 345 kV. Determine the sending end quantities, voltage regulation, and transmission efficiency.

5.7. Write a *MATLAB* function named **[ABCD] = abcdm(z, y, Lngt)** to evaluate and return the ABCD transmission matrix for a medium-length transmission line where **z** is the per phase series impedance per unit length, **y** is the shunt admittance per unit length, and **Lngt** is the line length. Then, write a program that uses the above function and computes the receiving end quantities, voltage regulation, and the line efficiency when sending end quantities are specified. The program should prompt for the following quantities:

The sending end line-to-line voltage magnitude in kV
The sending end voltage phase angle in degrees

The three-phase sending end real power in MW
The three-phase sending end reactive power in Mvar

Use your program to obtain the solution for the following case.

A three-phase transmission line has a per phase series impedance of $z = 0.03 + j0.4\ \Omega$ per km and a per phase shunt admittance of $y = j4.0 \times 10^{-6}$ siemens per km. The line is 125 km long. Obtain the ABCD transmission matrix. Determine the receiving end quantities, voltage regulation, and the line efficiency when the line is sending 407 MW, 7.833 Mvar at 350 kV.

5.8. Obtain the solution for Problems 5.8 through 5.13 using the **lineperf** program. Then, solve each problem using hand calculations.

A three-phase, 765-kV, 60-Hz transposed line is composed of four ACSR, 1,431,000-cmil, 45/7 Bobolink conductors per phase with flat horizontal spacing of 14 m. The conductors have a diameter of 3.625 cm and a GMR of 1.439 cm. The bundle spacing is 45 cm. The line is 400 km long, and for the purpose of this problem, a lossless line is assumed.

(a) Determine the transmission line surge impedance Z_c, phase constant β, wavelength λ, the surge impedance loading SIL, and the ABCD constant.
(b) The line delivers 2000 MVA at 0.8 lagging power factor at 735 kV. Determine the sending end quantities and voltage regulation.
(c) Determine the receiving end quantities when 1920 MW and 600 Mvar are being transmitted at 765 kV at the sending end.
(d) The line is terminated in a purely resistive load. Determine the sending end quantities and voltage regulation when the receiving end load resistance is 264.5 Ω at 735 kV.

5.9. The transmission line in Problem 5.8 is energized with 765 kV at the sending end when the load at the receiving end is removed.

(a) Find the receiving end voltage.
(b) Determine the reactance and the Mvar of a three-phase shunt reactor to be installed at the receiving end in order to limit the no-load receiving end voltage to 735 kV.

5.10. The transmission line in Problem 5.8 is energized with 765 kV at the sending end when a three-phase short-circuit occurs at the receiving end. Determine the receiving end current and the sending end current.

5.11. Shunt capacitors are installed at the receiving end to improve the line performance of Problem 5.8. The line delivers 2000 MVA, 0.8 lagging power factor. Determine the total Mvar and the capacitance per phase of the Y-connected capacitors to keep the receiving end voltage at 735 kV when the sending end voltage is 765 kV. *Hint:* Use (5.93) and (5.94) to compute the power angle δ and the receiving end reactive power. Find the sending end quantities and voltage regulation for the compensated line.

5.12. Series capacitors are installed at the midpoint of the line in Problem 5.8, providing 40 percent compensation. Determine the sending end quantities and the voltage regulation when the line delivers 2000 MVA at 0.8 lagging power factor at 735 kV.

5.13. Series capacitors are installed at the midpoint of the line in Problem 5.8, providing 40 percent compensation. In addition, shunt capacitors are installed at the receiving end. The line delivers 2000 MVA, 0.8 lagging power factor. Determine the total Mvar and the capacitance per phase of the series and shunt capacitors to keep the receiving end voltage at 735 kV when the sending end voltage is 765 kV. Find the sending end quantities and voltage regulation for the compensated line.

5.14. The transmission line in Problem 5.8 has a per phase resistance of 0.011 Ω per km. Using the **lineperf** program, perform the following analysis and present a summary of the calculation along with your conclusions and recommendations.

(a) Determine the sending end quantities for the specified receiving end quantities of $735\angle 0°$, 1600 MW, 1200 Mvar.
(b) Determine the receiving end quantities for the specified sending end quantities of $765\angle 0°$, 1920 MW, 600 Mvar.
(c) Determine the sending end quantities for a load impedance of $282.38 + j0\ \Omega$ at 735 kV.
(d) Find the receiving end voltage when the line is terminated in an open circuit and is energized with 765 kV at the sending end. Also, determine the reactance and the Mvar of a three-phase shunt reactor to be installed at the receiving end in order to limit the no-load receiving end voltage to 765 kV. Obtain the voltage profile for the uncompensated and the compensated line.
(e) Find the receiving end and the sending end current when the line is terminated in a three-phase short circuit.
(f) For the line loading of part (a), determine the Mvar and the capacitance of the shunt capacitors to be installed at the receiving end to keep the receiving end voltage at 735 kV when line is energized with 765 kV. Obtain the line performance of the compensated line.
(g) Determine the line performance when the line is compensated by series capacitor for 40 percent compensation with the load condition in part (a) at 735 kV.

(h) The line has 40 percent series capacitor compensation and supplies the load in part (a). Determine the Mvar and the capacitance of the shunt capacitors to be installed at the receiving end to keep the receiving end voltage at 735 kV when line is energized with 765 kV at the sending end.
(i) Obtain the receiving end circle diagram.
(j) Obtain the line voltage profile for a sending end voltage of 765 kV.
(k) Obtain the line loadability curves when the sending end voltage is 765 kV, and the receiving end voltage is 735 kV. The current-carrying capacity of the line is 5000 A per phase.

5.15. The ABCD constants of a lossless three-phase, 500-kV transmission line are

$$A = D = 0.86 + j0$$
$$B = 0 + j130.2$$
$$C = j0.002$$

(a) Obtain the sending end quantities and the voltage regulation when line delivers 1000 MVA at 0.8 lagging power factor at 500 kV.

To improve the line performance, series capacitors are installed at both ends in each phase of the transmission line. As a result of this, the compensated ABCD constants become

$$\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2}jX_c \\ 0 & 1 \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{2}jX_c \\ 0 & 1 \end{bmatrix}$$

where X_c is the total reactance of the series capacitor. If $X_c = 100\ \Omega$

(b) Determine the compensated ABCD constants.
(c) Determine the sending end quantities and the voltage regulation when line delivers 1000 MVA at 0.8 lagging power factor at 500 kV.

5.16. A three-phase 420-kV, 60-HZ transmission line is 463 km long and may be assumed lossless. The line is energized with 420 kV at the sending end. When the load at the receiving end is removed, the voltage at the receiving end is 700 kV, and the per phase sending end current is $646.6\angle 90°$ A.

(a) Find the phase constant β in radians per km and the surge impedance Z_c in Ω.
(b) Ideal reactors are to be installed at the receiving end to keep $|V_S| = |V_R| = 420$ kV when load is removed. Determine the reactance per phase and the required three-phase kvar.

5.17. A three-phase power of 3600 MW is to be transmitted via four identical 60-Hz transmission lines for a distance of 300 km. From a preliminary line